Moving layers and graded damage coupling with elasto-plasticity

Claude Stolz,

UMR9219 CNRS, Palaiseau, France, E-mail:claude.stolz@cnrs.fr

The paper investigate the coupling between graded damage [?] and elasto-plasticity. The elastic properties $\mathbb{C}$ of the material depends on a damage variable d, then the free energy w depends on the strain ε, on internal variables α and on damage d: $w(\varepsilon, \alpha, \mathrm{d})$.. Damage variable is bounded and its gradient is bounded by a concave function $f(\mathrm{d})$ in order to limit its concentration:

$$g_2(\mathrm{d}) = ||\nabla \mathrm{d}|| - \mathrm{f(d)} \leq 0, \quad \mathrm{f}(0) > 0.$$

These two conditions are taken into account by two Lagrange multipliers μ_i

$$\mu_i \geq 0, \quad g_1 = \mathrm{d}(\mathrm{d}-1) \leq 0, \quad \mathrm{g}_2 \leq 0,$$
$$\mu_1 g_1(\mathrm{d}) + \mu_2 \mathrm{g}_2(\mathrm{d}) = 0.$$

Introducing the potential energy $\mathcal{E}$ of a body Ω submitted to prescribed displacement on $\partial\Omega_u$ and tension on the complementary part $\partial\Omega_T$.

$$\mathcal{E}(u, \alpha, \mathrm{d}, \mu_\mathrm{i}) = \int_\Omega \mathrm{w}(\varepsilon(\mathrm{u}), \alpha, \mathrm{d})\mathrm{d}\Omega + \int_\Omega \mu_\mathrm{i}\mathrm{g}_\mathrm{i}(\mathrm{d})\mathrm{d}\Omega - \int_{\partial\Omega_\mathrm{t}} \mathrm{T}^\mathrm{d}.\mathrm{udS}$$

The evolution of the internal parameter (α, d) are given by normality laws

$$\dot{\alpha} = \lambda\frac{\partial \Phi}{\partial A}, \lambda \geq 0, \Phi(A) \leq 0, \lambda\Phi = 0 \quad \dot{\mathrm{d}} \geq 0, Y - Y_c \leq 0, \dot{\mathrm{d}}(Y - Y_c) = 0$$

where Φ is a convex function of the thermodynamical force A associated to α :$A = -\dfrac{\partial \mathcal{E}}{\partial \alpha}$, and Y_c is a critical value for local fracture, Y is the release rate of energy associated to damage evolution, $Y = -\dfrac{\partial \mathcal{E}}{\partial \mathrm{d}}$.
The variations of potential energy gives the driven forces:

$$A = -\frac{\partial w}{\partial \alpha}, \quad Y = -\frac{\partial w}{\partial \mathrm{d}} + \frac{1}{f}\, div(\mu_2 \nabla \mathrm{d}) + \mu_1(1 - 2\mathrm{d})$$

Variations of potential energy exhibit discontinuities along the boundary between sound and damaged material, this fact must be discuss.
In particular, w can be discontinuous along moving boundaries, especially along the surface where $\mathrm{d} = 0^+$. If such a discontinuity exists additional dissipation occurs, if not this imposes some continuity conditions on the internal variable α. In this case, in the damaged zone plasticity and damage cannot evolve simultaneously as shown in [?].
This fact is illustrated on analytical examples based on cylindrical or spherical geometries on elasto-plasticity with or without linear hardening.

References

[1] C. Stolz, *On damage regularity defect nucleation modelling*, Int. Journal of Solids and Structures, 229 : 111-107, (2022).

[2] R. M. Pradeilles-Duval, C. Stolz, *Mechanical transformation and discontinuities along a moving surface*, J. Mech. Phys. Solids,43(1) : 91-12, (1995).

[3] N. Moës, C. Stolz, P.E. Bernard, N. Chevaugeon, ,, A level set based model for damage growth: the thick level set approach, Int. J. Numer. Meth. Engng. : 86(3), (358-380). 2011

[4] N. Valorozo, C. , Stolz, *Graded damage in quasi-brittle solids*, Int. J. Numer. Meth. Engng.,123(11) : 2467-2498, (2022).