Numerical solutions for non-linear mechanics using the Asymptotic Numerical Method (ANM)

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Summary

Many problems of physics and technology lead to non-linear equations depending on at least one scalar parameter λ. They can be written as: $R(U, \lambda) = 0$, $U \in \mathbb{R}^N$, $R \in \mathbb{R}^N$. The aim of this course is to present a family of numerical methods, called Asymptotic Numerical Methods (ANM) [1], to compute the response curves (or solution branches) $U(\lambda)$ of the non-linear equations $R(U, \lambda) = 0$. These ANM rely on the calculation of Taylor series, and they can be seen as an alternative to the classical continuation methods based on prediction-correction algorithms.

The ANM chains several calculations, as with many other algorithms, but here one computes a sequence of "pieces" of solution branches of the equation $R(U, \lambda) = 0$: piece^1, piece^2, piece^3 · · · piece^j, piece^{j+1} · · · One generates a sequence of pieces of branches, each new piece being defined by a Taylor series computed from a given starting point.

In this course, we will apply the ANM to non-linear elasticity. At small deformations, the behavior of solids is reversible, and a linear behavior law is sufficient, but geometric non-linearities are often considered in the case of thin structures such as beams, plates and shells.

Hyper-elastic materials exhibit reversible behavior at very large deformations. Rubber-like materials, for example, combine a geometrically exact framework with non-linear stress-strain laws. Today, hyper-elasticity is used to model biological materials. A major concern in biomechanical research is the effect of tissue growth. In this course, we discuss the application of Taylor series to the most common hyper-elastic models.

[1] B. Cochelin, N. Damil, M. Potier-Ferry, Méthode Asymptotique Numérique, Hermès Lavoisier, 2007.